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THE ORDER OF NUMBERS AND THE GOLDBACH CONJECTURE

Jacqueline Wötzel

ABSTRACT

Following will be regard the potentiality of order of numbers for the ternary Goldbach conjecture, where he claimed that "every number… is an aggregate of three prime numbers". The order of numbers illustrates the possible combinations of prime numbers for the generation of all natural (integer) numbers. Two ways to fragment a natural number in a sum of prime numbers are represented. The ternary conjecture of Goldbach can be confirmed at this way. A binary addition is also possible and demands that one summand be a product with a prime number.

KEYWORDS

Number theory, Order of integer numbers, Goldbach conjecture

1. INTRODUCTION

Goldbach discussed in a letter to Euler the way of Fermat to generate prime numbers and he was searching for possibilities to fragment natural numbers in a sum of prime numbers.[1] With the discovered order of numbers based on a partition technique of all subsequent numbers in three classes, it is possible to find a fast and efficient way for the dissection of every number in one included highest prime number and a remained number.[2] This article will investigate the fragmentation of natural numbers in a ternary sum of prime numbers or as sum of a prime number and a product with prime. Another way is to use the order of numbers, where in column 1 are to find all prime numbers additional to the multiple of 5 and 7. This two ways are possible to reconsider the Goldbach conjecture.

2. ORDER OF INTEGER NUMBERS AND THE GOLDBACH CONJECTURE

2.1. Order of natural numbers

Based on a simple partition technique all the two and three divisible numbers appear in separate columns, whereby all prime numbers are contained in the resulted first column. This ascending order allows endless generation or identification of affiliation to the three columns or classes. The successional natural numbers are separated in ascending fashion in three columns with given rule. The numbers 1, 2 and 3 are the header and starting point for the three main-columns. To build up, it is useful to start with the column 3 where all three-divisible numbers are placed. In column 2 are to found all two-divisible numbers before this multiple of 3. In column 1 are placed the other numbers before the used multiple of 3, see table 1. All prime numbers and some multiples of 5 and 7 are in column 1. The multiples of 5 and 7 are to find in all three columns with an alternating regular distance, see table 2. The successive numbers can be separate endless adequate to this properties and this allows to identify a structure of the natural and also integer numbers. [2]

Table 1. Cutout of separation natural numbers at three columns with 1, 2 and 3 as head.

1	**2**	**3**
5	4	6
7	8	9
11	10	12
13	14	15
17	16	18
…	…	…

[2]

All three columns are crossed by multiples of 5 and 7 whose pattern is visible with the representation in table 2, where the separate quotients of 5 and 7 are related to the numbers of column 1. The quotients of 5 and 7 are able to appear as double helices with defined distances and an endless pattern. To recognize helices, the first double helices are marked in orange. Thus allowed to find equidistant elimination steps for the multiples of 5 and 7 in the three columns. [2]

Table 2. Cutout of all quotients to five and seven in the three columns – separated.

Column 1	All multiples of 5 in the three columns			All multiples of 7 in the three columns		
	1	2	3	1	2	3
1						
5	1					
7				1		
11		2				
13			3		2	
17						
19		4				3
23						
25	5					
29			6		4	
31						
35	7			5		
37						
41		8				6
43			9			
47						
49		10		7		
53						
55	11				8	
59			12			
61						9

[2]

2.2. Relevance of order of numbers to the Goldbach conjecture

All prime numbers can be identify by this order of numbers, see table 1 considering the pattern of multiples of 5 and 7, see table 2. The multiples of 5 and 7 can be superposed, e.g. for 35, or they can be neighbored. There are two cases to appear of neighboring of multiples of 5 and 7 in column 1, see red marked numbers in cutouts in table 3 and 4. The two cases of neighboring multiples helps to identify the difference from an arbitrary number to the nearest smaller prime number in column 1. The two cases are to find reiterative endless.

89	88	90
91	92	93
95	94	96

Table 3. Case 1 for proximity of multiples of 5 and 7 in column 1

199	200	201
203	202	204
205	206	207

Table 4. Case 2 for proximity of multiples of 5 and 7 in column 1

For table 3 as example is 7 the maximum difference from prime number 89 to all successive numbers inclusive 96. The number 88 is smaller as 89 and would be have the difference of 5 to the next smaller prime number. For table 4 as example is 8 the maximum difference from prime number 199 to all successive numbers inclusive 207. This differences also have a pattern in there sequence.

2.3. Partition technique with 3 summands of prim

All natural numbers can be composed of the sum of prim and a number between 0 and 8, see equation 1.

$$\boxed{n = k + p}$$
$$n \in \mathbb{N} \mid 1 \leq n \leq \infty,$$
$$k \in \mathbb{N} \mid k \in \{0,1,2,3,4,5,6,7,8\},$$
$$p \in \mathbb{P} \qquad \text{(eq.1)}$$

Three variations are possible; variation 1: a natural number is a prime number (k=0), variation 2: a natural number is a prime number to sum with k=1,3,5 or 7 and variation 3: a natural number is a prime number to sum with k=2,4,6 or 8. The even numbers in variation 3 are representable as sum of 1 and the numbers 1,3,5 or 7 or a product of 2 and multiples, see equation 2.

$$\boxed{k = 2^m r = q + 1}$$

$$k \in \mathbb{N} \mid k \in \{2,4,6,8\},$$
$$m \in \mathbb{N} \mid m \in \{1,2,3\},$$
$$r \in \mathbb{P} \mid r \in \{1,3\},$$

$$q \in \mathbb{P} \mid q \in \{1,3,5,7\} \qquad \text{(eq.2)}$$

The formation of every number is therefore possible binary, where one summand is a prime number and one summand is a product of multiples of two (2^m , m∈ N | m ∈{1,2,3}) with the prime numbers 1 or 3. The formation of every number is also possible as sum of maximal three prime numbers. In this case the first summand is the largest prime number underneath a arbitrary natural number, the second summand can be 1,3,5 or 7 and the third summand will be 1.

2.4. Prim and the role of number 1

The role of the number 1 is in discussion, because of Eulers fundamental book with instructions to Algebra. In Chapter 4 – the original is written in an old German language and printed in old lettering – corresponding called “About the nature of integer numbers in dependent of whose role as factors” he aims to show the numbers in their function as factors. He makes the difference between numbers who can build up by factors and the other group, which can`t build up by factors, which he calls “simple” numbers or prime numbers. His explanation starts by 2 because 1 as factor can`t change numbers.[3] At this way he avoids the trivial redundancy of implication of number 1 as factor for prime numbers, because it has no function. The definition of prime numbers allows to see the number 1 as a prime number. As divisible by 1 and itself it is the special case of the same dividend, divisor and quotient. In the scheme of order of numbers, see table 1 the numbers 1,2,3,5 and 7 have also a special function as head of columns, whereby the columns of 5 and 7 permeate the columns of 1,2 and 3. The number 1 is the head and starting point and it is important to start there in column 1 where are included all prime numbers in a simple pattern.

2.5. Partition technique with 2 summands where one is a product of prim

Another way to part a natural number is about the periodicity of numbers in column 1, see table 5. After divide all numbers of column 1 by 9, is to find a high periodicity in consecutive decimal numbers. With summation of number 2 and multiples to the sequential numbers they are to identify everywhere.

Table 5: Periodicity in column 1 after divided by 9

Numbers of column 1 divided by 9
0,11111111
0,55555555
0,77777777
1,22222222
1,44444444
1,88888888
….

The arbitrary number is to divide by 9 to see the distance of this number to column 1 which can be 0, 0.**1**, 0.**2** or 0.**3** and will be one of the summand 0,1,2,3. At this way every number can be descriptive by two summands, whereby one is number 0,1,2 or 3 and the second a factor of prim with 1,5,7 or 35 and multiples, see equation 3. In the structure of all quotients in column 1is to find the same sequential regularity like in column 1, where 4/2 is the answer of the question to the distances. The multidimensionality of order of numbers is visible.

$$\boxed{n = a + kb^m p}$$

$$n,k,m \in \mathbb{N} \mid 1 \leq n,k,m \leq \infty ,$$
$$a \in \mathbb{N} \mid a \in \{0,1,2,3\},$$
$$b \in \mathbb{N} \mid b \in \{1,5,7,35\},$$
$$p \in \mathbb{P} \qquad \text{(eq.3)}$$

3. Conclusions

Every number is representable as the sum of maximum three prime numbers, where the first summand is the largest prime number underneath the given number, the second summand can be 1,3,5 or 7 and the third summand will be 1. Every number is also representable as the sum of maximum two summands, where one summand will be a prime number and the second summand is a product of the factor 1,2,5,7 or 35 and there multiples with a prime number. The first Goldbach conjecture, also called “ternary” can be confirmed with the consideration of discovered order of numbers. A binary addition is also possible and demands that one summand is a product with a prime number. Two ways are represented to fragment a natural number in a sum of prime numbers. The rules are also applicable for integer numbers.

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I appreciate my family!

References

[1] Letter of Goldbach to Euler from 7 June 1742 in Correspondance mathématique et physique de quelques célèbres géomètres du XVIIIème siècle (Band 1), St.-Pétersbourg 1843, pp. 127

[2] Jacqueline Wötzel,(2023) Order of integer numbers and structure of prime numbers, Inter-national Journal on Computational Science & Applications (ILJSCA) Vol.13, No.1, pp1-2

[3] Leonhard Euler, Vollständige Anleitung zur Algebra, Erster Theil von den verschiedenen Rechnungsarten, Verhältnissen und Proportionen, St.Petersburg 1771 bey der Kayserlichen Akademie der Wissenschaften, Seite 16-17

Author

Jacqueline Wötzel is a PhD for polymer engineering and has studied at the Martin-Luther-University Halle-Wittenberg, Germany. She worked for different research and development departments and now as consultant. She lives in the East of Germany and her interest are structures.